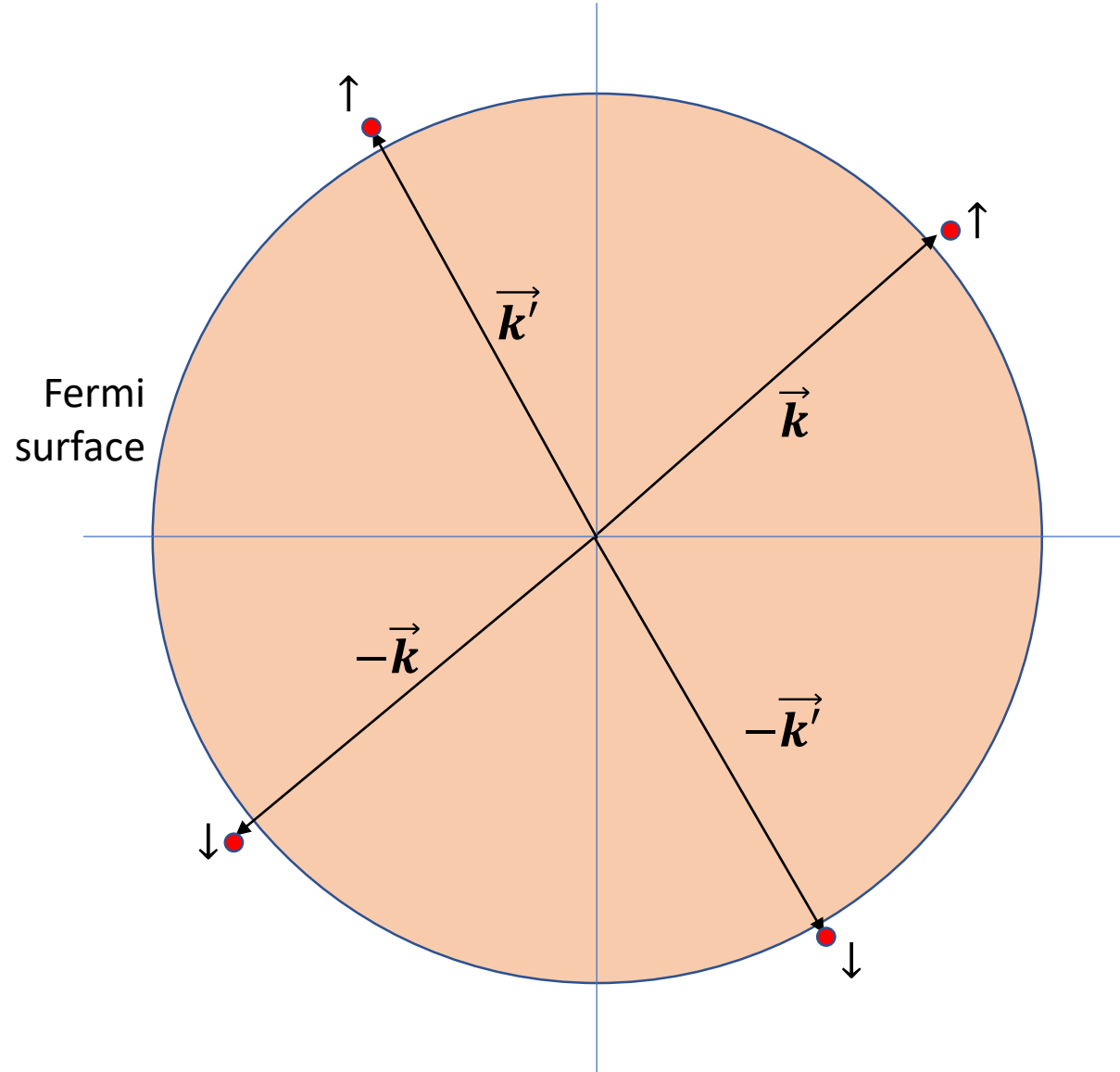
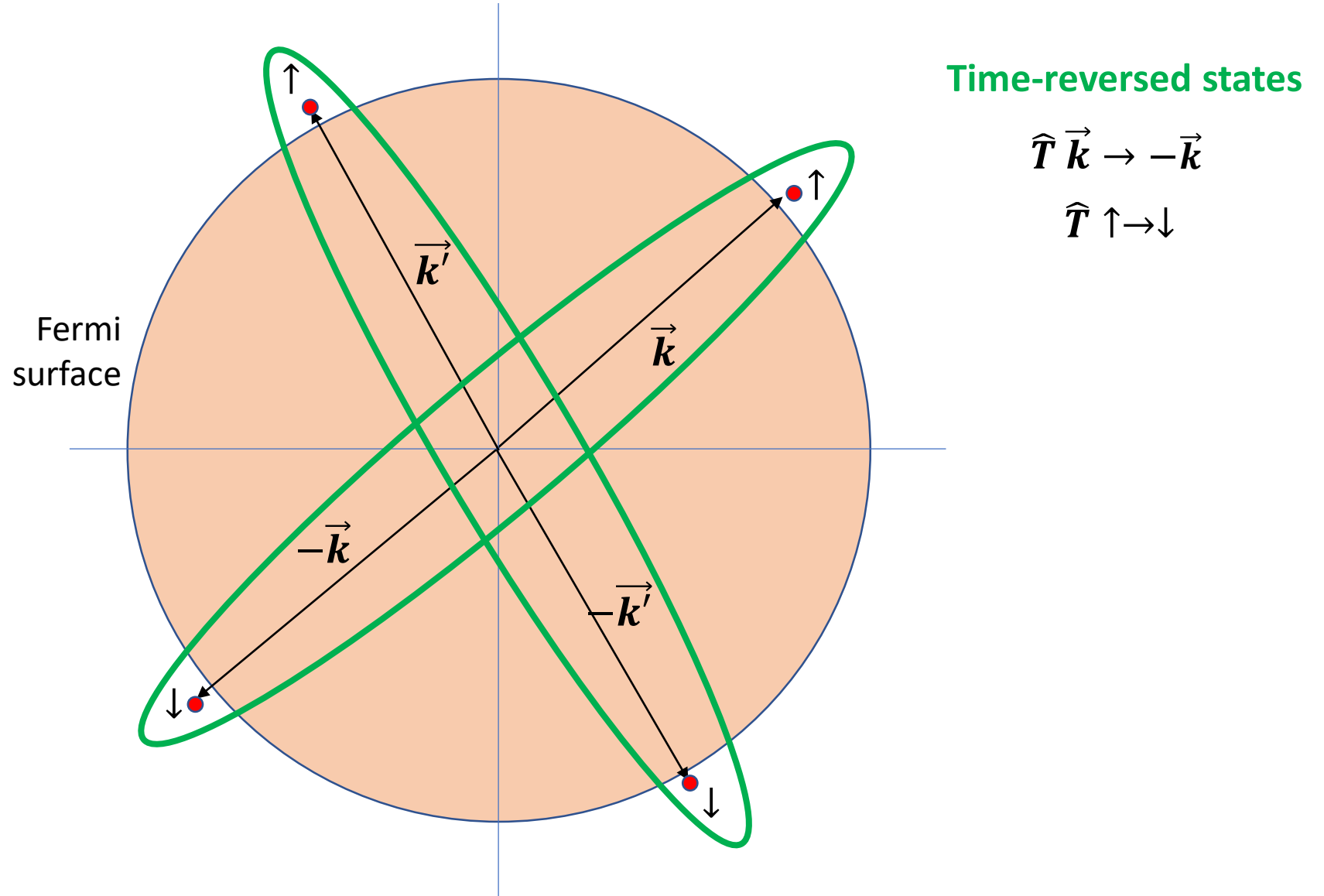


Two Cooper Pairs

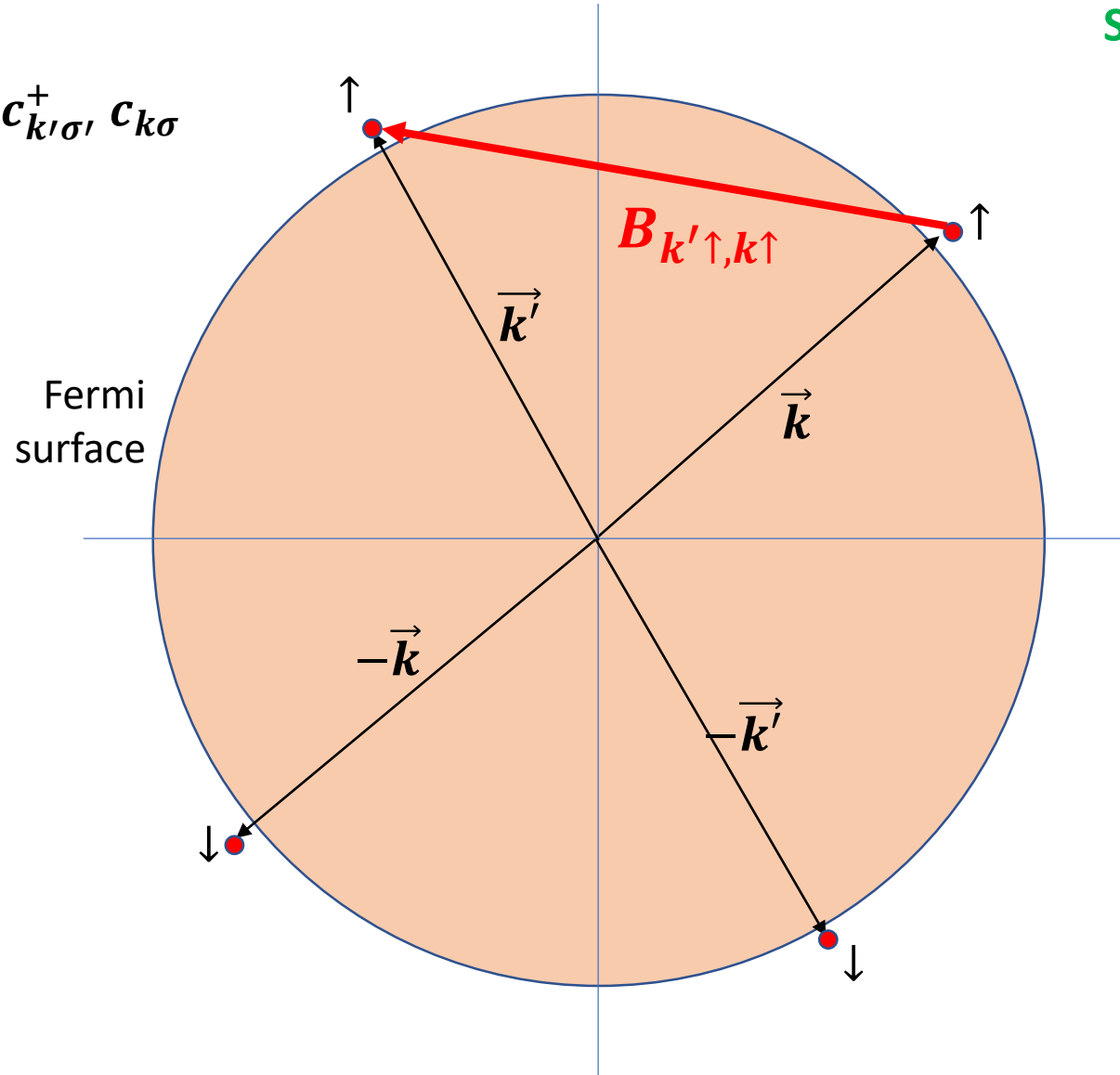


Pairing of Time-Reversed States



Scattering Between Single-Particle States due to Perturbation (I)

$$\mathcal{H}_{pert} = \sum_{k\sigma, k'\sigma'} B_{k'\sigma', k\sigma} c_{k'\sigma'}^\dagger c_{k\sigma}$$



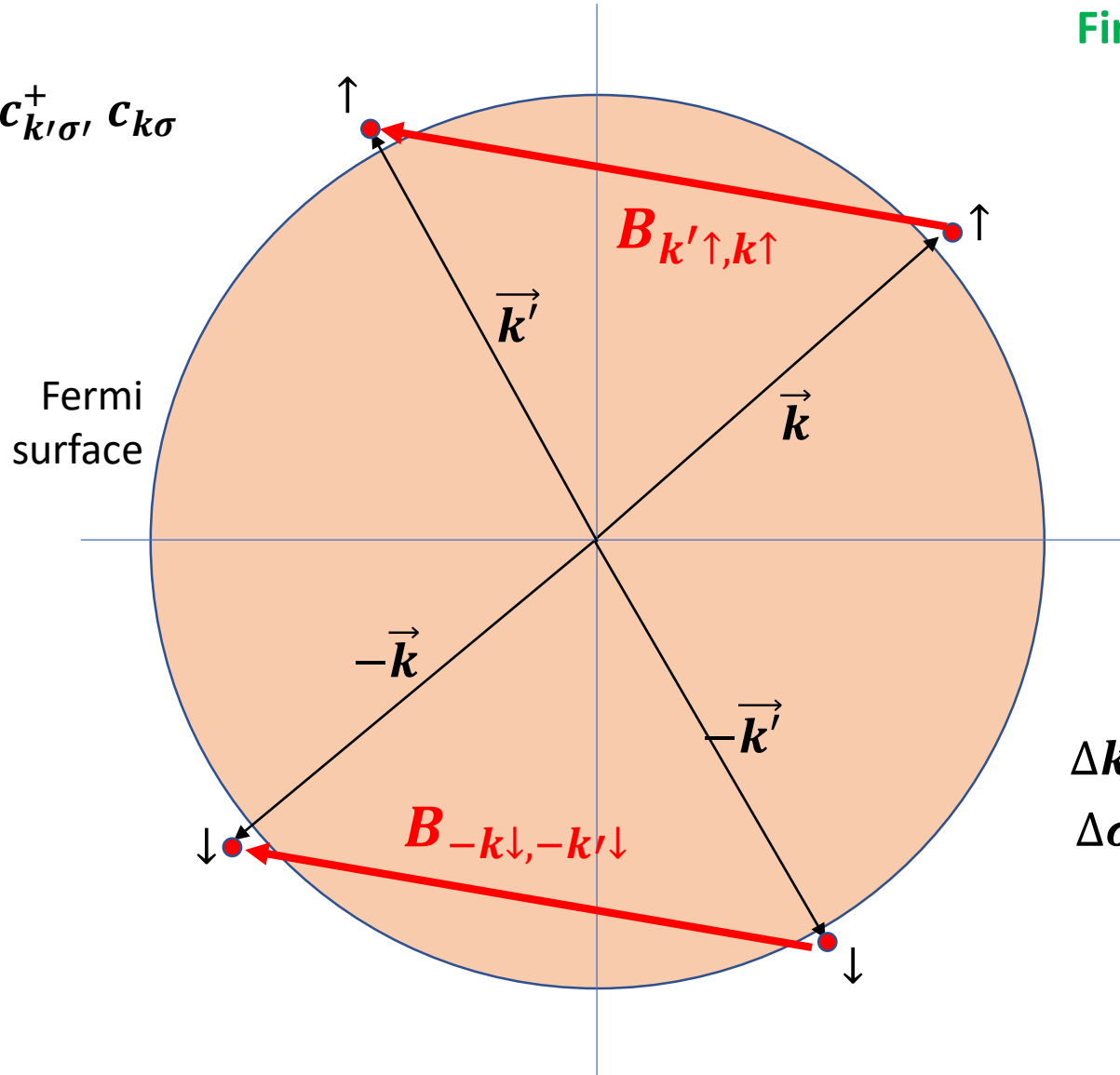
Scattering event

$$\Delta k = k' - k$$

$$\Delta \sigma = \sigma' - \sigma$$

Scattering Between Single-Particle States due to Perturbation (II)

$$\mathcal{H}_{pert} = \sum_{k\sigma, k'\sigma'} B_{k'\sigma', k\sigma} c_{k'\sigma'}^+ c_{k\sigma}$$



First scattering event

$$\Delta k = k' - k$$

$$\Delta \sigma = \sigma' - \sigma$$

Consider another term
in the sum ...

Second scattering event

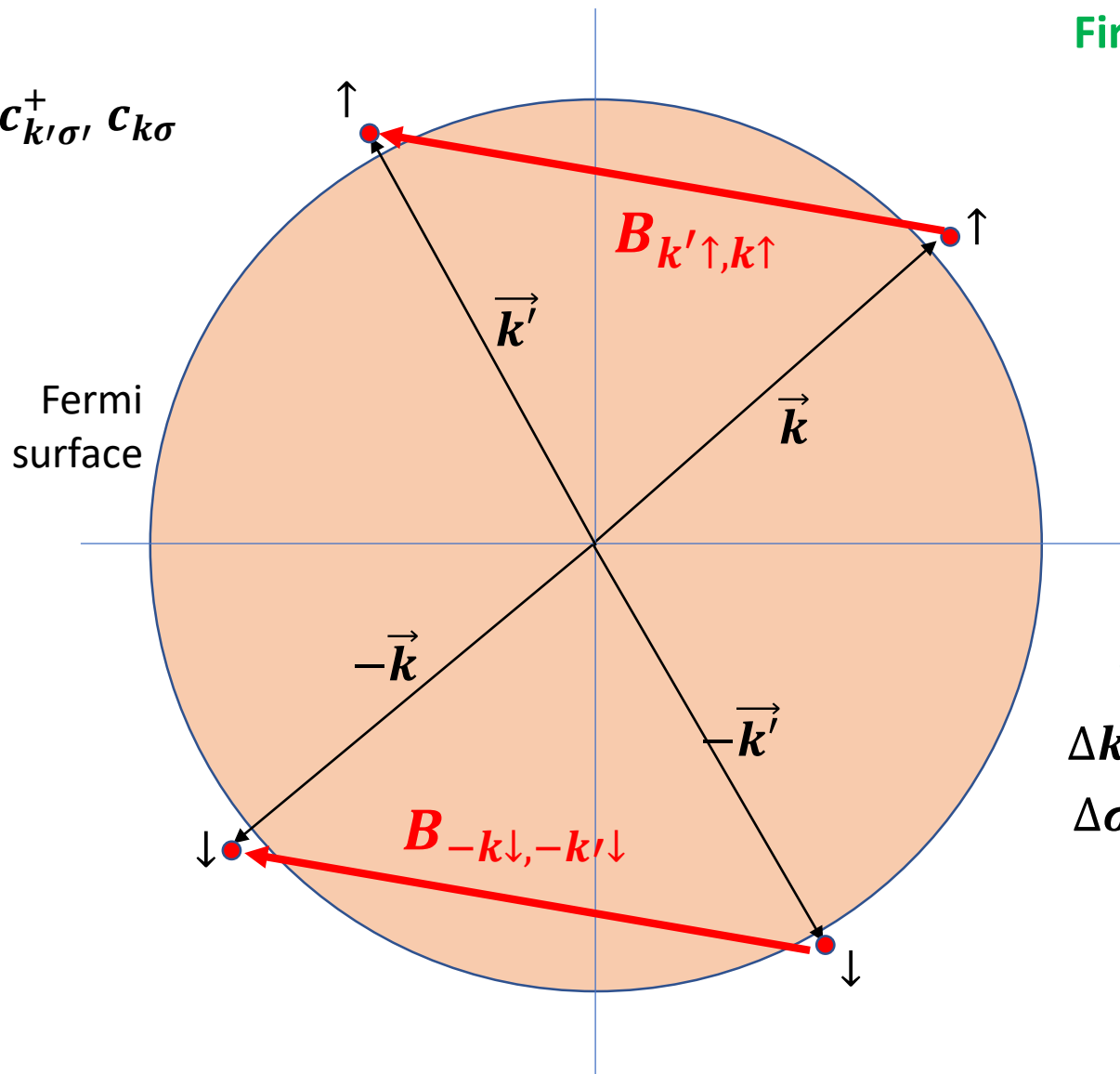
$$\Delta k = -k - (-k') = k' - k$$

$$\Delta \sigma = -\sigma - (-\sigma') = \sigma' - \sigma$$

The same momentum transfer and the same change in spin state occurs,
due to the correlated nature of the Cooper pairs in the original states

Scattering Between Single-Particle States due to Perturbation (III)

$$\mathcal{H}_{pert} = \sum_{k\sigma, k'\sigma'} B_{k'\sigma', k\sigma} c_{k'\sigma'}^+ c_{k\sigma}$$



First scattering event

$$\Delta k = k' - k$$

$$\Delta \sigma = \sigma' - \sigma$$

Second scattering event

$$\Delta k = -k - (-k') = k' - k$$

$$\Delta \sigma = -\sigma - (-\sigma') = \sigma' - \sigma$$

These two matrix elements can differ at most by a sign: $B_{k'\sigma', k\sigma} = \pm B_{-k-\sigma, -k'-\sigma'}$

depending on whether the perturbation is symmetric (case I) or anti-symmetric (case II) upon time-reversal of the electronic state

Scattering Between Single-Particle States due to Perturbation (IV)

$$\begin{aligned}\mathcal{H}_{pert} &= \sum_{k\sigma, k'\sigma'} B_{k'\sigma', k\sigma} c_{k'\sigma'}^+ c_{k\sigma} \\ &= B_{k'\sigma', k\sigma} (c_{k'\sigma'}^+ c_{k\sigma} \pm c_{-k-\sigma}^+ c_{-k'-\sigma'}) + \dots\end{aligned}$$

Case I (+): Ultrasonic attenuation, ...

Case II (-): Electromagnetic absorption, nuclear spin relaxation, ...

These groups of terms give rise to “coherence factors” in the calculation of absorption rates, etc.